Published by Institute of Physics Publishing for SISSA/ISAS
Received: *June 11, 2004*
Revised: *November 15, 2004*
Accepted: *November 15, 2004*

# Ricci-flat deformation of orbifolds and localized tachyonic modes

**Yosuke Imamura, Fumikazu Koyama and Ryuji Nobuyama**

*Department of Physics, University of Tokyo*
*Hongo, Tokyo 113-0033, Japan*
*E-mail:* imamura@hep-th.phys.s.u-tokyo.ac.jp, koyama@hep-th.phys.s.u-tokyo.ac.jp, nobuyama@hep-th.phys.s.u-tokyo.ac.jp

Abstract: We study Ricci-flat deformations of orbifolds in type II theory. We obtain a simple formula for mass corrections to the twisted modes due to the deformations, and apply it to originally tachyonic and massless states in several examples. In the case of supersymmetric orbifolds, we find that tachyonic states appear when the deformation breaks all the supersymmetries. We also study nonsupersymmetric orbifolds $\mathbb{C}^2/\mathbb{Z}_{2N(2N+1)}$, which is T-dual to $N$ type 0 NS5-branes. For $N \geq 2$, we compute mass corrections for states, which have string scale tachyonic masses. We find that the corrected masses coincide to ones obtained by solving the wave equation for the tachyon field in the smeared type 0 NS5-brane background geometry. For $N = 1$, we show that the unstable mode representing the bubble creation is the unique tachyonic mode.

Keywords: String Duality, Tachyon Condensation.

## Contents



## 1. Introduction

Tachyon condensation is one of main topics in string theory actively investigated in the recent years. The connection between open string tachyons and the instability of D-branes conjectured by Sen [1, 2, 3, 4] has made great progress in our understanding of dynamics of D-branes.

On the other hand we know much less about the closed string tachyon condensation. One of the reasons is the close connection between closed strings and the spacetime. We would expect some drastic change of the background spacetime itself in the process of condensation. Therefore it seems difficult to analyze the closed tachyon condensation using conventional ways. However, if closed tachyons exist only as localized states, we can discuss more or less the fate of the spacetime when tachyons condensate.

Several types of closed tachyons have been discussed. In [5, 6], relation between instability of fluxbranes and closed string tachyon condensation is discussed. In [7], it is suggested that condensation of tachyonic twisted modes localized at fixed points of non-supersymmetric orbifolds like $\mathbb{C}/\mathbb{Z}_n$ and $\mathbb{C}^2/\mathbb{Z}_n$ give rise to deformations of geometry of the background spacetimes. Configurations bringing with localized tachyons are also discussed in many works [8]–[20]. Recently, the potentials of localized tachyons has been computed by string field theory [21].

In general, tachyonic twisted modes at an orbifold fixed point have tachyonic masses of the order of the string mass scale $\alpha'^{-1/2}$. This implies that we cannot directly analyze the decay process itself associated with these tachyons without the help of string theory. If we have localized tachyons with milder tachyonic masses which does not include the string



JHEP11(2004)042

mass scale, it may be possible to treat the condensation process in some classical way. To obtain such modes, let us consider an orbifold with massless twisted modes, and perturb the background geometry so that it has non-vanishing curvature of the order of $1/L^2$ with $L$ much larger than $\alpha'^{1/2}$. Then we expect the twisted modes, which were originally massless, receive mass corrections $\delta M^2 \sim 1/L^2$, and it can be negative depending on the way of deformation. If $L$ is sufficiently large, it is expected for us to be able to analyze this instability with classical (super)gravity.

In this paper we demonstrate that we can actually obtain twisted states with small tachyonic masses in this way. We consider several abelian orbifolds $\mathbb{R}^{10-d} \times \mathbb{R}^d/\mathbb{Z}_n$ with $d = 4, 6$ and 8, and discuss the deformation of the metric of the $\mathbb{R}^d/\mathbb{Z}_n$ factor. Other fields (dilaton, $B$-field, metric of $\mathbb{R}^{10-d}$, etc.) will be kept intact. The equation of motion for the metric demands that the deformed background is Ricci-flat. This also guarantees the conformal symmetry on worldsheets. We compute the mass correction to twisted modes with the help of worldsheet non-linear $\sigma$-model.

In [22], the T-duality between type 0 NS5-branes and an orbifold deformed in this way is discussed, and the existence of a tachyonic mode responsible to the creation of Witten's bubble are shown. However, it is difficult to find all the unstable modes and to determine their tachyonic masses by analyzing small fluctuations of the metric. Indeed only the existence of at least one tachyonic mode was shown in [22], and the number of all localized tachyons and their masses including numerical factors were not determined. One purpose of this paper is to give an answer to this problem with the help of string theory. Although the small tachyonic mass enables us to use classical gravity, and the geometric interpretation is clarified, string theory is still a powerful tool to compute the masses of the tachyonic modes because the correspondence between massless twisted modes of supersymmetric orbifolds and geometric deformations of the geometry has been well understood [23].

This paper is organized as follows. In the next section, we obtain a general formula for the mass correction. We apply the formula to several examples of supersymmetric orbifolds in section 3. We see that we can obtain tachyonic modes when the deformation breaks all the supersymmetry. In section 4, we discuss Taub-NUT deformation of non-supersymmetric orbifolds, which is denoted by $\mathbb{C}^2/\mathbb{Z}_{2N(2N+1)}$ with the notation in [7], and their relations to type 0 NS5-branes. The last section is devoted to conclusions and discussions.

## 2. Ricci-flat deformation

Type II strings on curved backgrounds are described by the supersymmetric non-linear $\sigma$-model

$$L = -\frac{1}{4\pi\alpha'}\int_0^{2\pi} d\sigma \Big[G_{mn}(\Phi)(\partial_\mu\Phi^m\partial^\mu\Phi^n + i\Psi^m\gamma^\mu D_\mu\Psi^n) + \\ + \frac{1}{6}R_{mpnq}(\Phi)(\Psi^m\Psi^n)(\Psi^p\Psi^q)\Big], \tag{2.1}$$

where $\Phi^m$ are scalar fields representing the coordinates in the target space, and their superpartners $\Psi^m$ are two-component Majorana spinors. The metric $G_{mn}(\Phi)$ and the curvature tensor $R_{mnpq}(\Phi)$ depend on the coordinates $\Phi^m$. In order not to spoil the conformal invariance, the background must be Ricci-flat. Because of the Ricci-flatness of the background spacetime, the curvature tensor $R_{mnpq}$ is identical to the Weyl tensor. It is known that the number of independent components of the Weyl tensor in $d$-dimension ($d \geq 3$) is $d(d+1)(d+2)(d-3)/12$.

We treat the background curvature as perturbations to flat orbifolds, and our aim in this section is to give a general formula for the mass corrections of twisted modes up to first order in the curvature. First task we have to do is to expand the action (2.1) as $L = L_{(0)} + L_{(1)} + \cdots$, where $L_{(0)}$ and $L_{(1)}$ are the free part and the leading interaction terms proportional to the curvature tensor, respectively. The ellipsis represent higher order terms, which we are not interested in here. The free equations of motion for $\Phi^m$ and $\Psi^m$ obtained from $L_{(0)}$ are solved by

$$\Phi^m(\sigma^+,\sigma^-) = \phi^m(\sigma^+) + \widetilde{\phi}^m(\sigma^-), \qquad \Psi^m(\sigma^+,\sigma^-) = \begin{pmatrix} \psi^m(\sigma^+) \\ \widetilde{\psi}^m(\sigma^-) \end{pmatrix}. \tag{2.2}$$

We consider twisted modes with twisted boundary conditions

$$\Phi^m(\sigma+2\pi) = U\Phi^m(\sigma)U^{-1}, \qquad \Psi^m(\sigma+2\pi) = U\Psi^m(\sigma)U^{-1}, \tag{2.3}$$

where $U$ is an element of the orbifold group. The OPEs for the left moving parts $\phi^m$ and $\psi^m$ are

$$\phi^m(\sigma)\partial_+\phi^n(0) = \frac{\alpha' G^{mn}}{2\sigma} + \text{regular}, \qquad \psi^m(\sigma)\psi^n(0) = \frac{-i\alpha' G^{mn}}{\sigma} + \text{regular}. \tag{2.4}$$

We also have similar OPEs for $\widetilde{\phi}^m$ and $\widetilde{\psi}^m$. From these OPEs and the boundary conditions (2.3), we can determine the propagators including the regular terms unambiguously.

The interaction term $H_{\text{int}} = -L_{(1)}$ in the hamiltonian is obtained by the normal coordinate expansion of the action (2.1).

$$\begin{aligned} H_{\text{int}} = -L_{(1)} = \int_0^{2\pi} \frac{d\sigma}{2\pi\alpha'} \Big[ & -\frac{1}{6} R_{mpnq} \Phi^p \Phi^q \partial_\mu \Phi^m \partial^\mu \Phi^n - \\ & - \frac{i}{6} R_{mpnq} \Phi^p \Phi^q (\Psi^m \gamma^\mu \partial_\mu \Psi^n) + \frac{i}{4} R_{pqmn} (\Phi^p \partial_\mu \Phi^q)(\Psi^m \gamma^\mu \Psi^n) + \\ & + \frac{1}{12} R_{mpnq} (\Psi^m \Psi^n)(\Psi^p \Psi^q) \Big]. \end{aligned} \tag{2.5}$$

$R_{mnpq}$ in (2.5) denotes the curvature tensor at the orbifold fixed point $\Phi^m = 0$. Note that the operator products in (2.5) are not singular at the contact limit. As we see in (2.4), the singular parts of OPEs are proportional to the inverse metric $G^{mn}$. Because of the Ricci-flatness of the background, this singularity does not contribute to $H_{\text{int}}$.

Let $J^{mn}$ denote anti-hermitian currents associated with the targetspace rotational symmetry, $K^{mn}$ and $\widetilde{K}^{mn}$ its left and right moving parts, respectively. They are given by

$$K^{mn} = \frac{1}{\alpha'}\left(2i\phi^{[m}\partial_+\phi^{n]} + \psi^{[m}\psi^{n]}\right), \qquad \widetilde{K}^{mn} = \frac{1}{\alpha'}\left(2i\widetilde{\phi}^{[m}\partial_-\widetilde{\phi}^{n]} + \widetilde{\psi}^{[m}\widetilde{\psi}^{n]}\right). \tag{2.6}$$



JHEP11(2004)042

By substituting the solution (2.2) to (2.5), we obtain the following expression with the help of the Bianchi identity.

$$\begin{aligned} H_{\text{int}} &= -\frac{\alpha'}{4}\int_0^{2\pi}\frac{d\sigma}{2\pi}R_{mnpq}K^{mn}\widetilde{K}^{pq} + H' \\ &= -\frac{\alpha'}{4}\sum_k R_{mnpq}K_k^{mn}\widetilde{K}_k^{pq} + H' \,, \end{aligned} \tag{2.7}$$

where $K_k^{mn}$ and $\widetilde{K}_k^{mn}$ are Fourier modes of $K^{mn}$ and $\widetilde{K}^{mn}$. $H'$ is defined by

$$\begin{aligned} H' = \int_0^{2\pi}\frac{d\sigma}{2\pi\alpha'}\Big[ &-\frac{i}{2}R_{pqmn}(\widetilde{\phi}^p\widetilde{\psi}^m\widetilde{\psi}^n)(\partial_+\phi^q) - \frac{i}{2}R_{pqmn}(\phi^p\psi^m\psi^n)(\partial_-\widetilde{\phi}^q) + \\ &+\frac{2}{3}R_{mpnq}(\widetilde{\phi}^p\widetilde{\phi}^q\partial_-\widetilde{\phi}^n)(\partial_+\phi^m) + \frac{2}{3}R_{mpnq}(\phi^p\phi^q\partial_+\phi^m)(\partial_-\widetilde{\phi}^n) + \\ &+\frac{1}{6}R_{mpqn}\partial_+(\phi^m\phi^n)\partial_-(\widetilde{\phi}^p\widetilde{\phi}^q)\Big] \,. \end{aligned} \tag{2.8}$$

All the terms in $H'$ are in fact irrelevant to our computation of the mass correction by the following reason. Let us look at the first term in $H'$. It is a product of the left moving part $\partial_+\phi^q$ and the right moving part $\widetilde{\phi}^p\widetilde{\psi}^m\widetilde{\psi}^n$. We are interested in the leading term in the perturbative expansion of the mass correction. At this level of perturbation, we need only the energy-preserving matrix elements of the interaction operator $H_{\text{int}}$. However, because of the derivative in the left moving factor $\partial_+\phi^q$, this term does not have energy-preserving matrix elements. This is the case for any term in $H'$. Thus $H'$ is irrelevant to our computation.

Furthermore, because both the operators $K_k^{mn}$ and $\widetilde{K}_k^{mn}$ change the worldsheet energy $L_0 + \widetilde{L}_0$ by $k$, only the zero mode with $k = 0$ has energy preserving parts contributing to the mass correction.

Finally we obtain a quite simple formula for the interaction term relevant to the mass correction.

$$H_{\text{int}} = -\frac{\alpha'}{4}R_{mnpq}K_0^{mn}\widetilde{K}_0^{pq} \,. \tag{2.9}$$

Note that the Fourier index $k$ of $K_k^{mn}$ and $\widetilde{K}_k^{mn}$ is not always an integer because the currents $J^{mn}$ satisfy the twisted boundary condition $J^{mn}(\sigma + 2\pi) = UJ^{mn}(\sigma)U^{-1}$. Only generators commuting with $U$ have zero modes and contribute to the mass correction.

## 3. Deformation of supersymmetric orbifolds

In this and the next sections, we use the formula (2.9) to compute the mass corrections to the twisted modes in several orbifolds. Although the formula is quite general and we can in principle compute the mass correction for any states in twisted sectors, we focus on low-lying states in the NS-NS sector because we are interested in the geometric instability of the orbifold.

In this section, we discuss deformation of supersymmetric orbifolds.

JHEP11(2004)042

### 3.1 $\mathbb{C}^2/\mathbb{Z}_{N(1)}$

First, we consider supersymmetric $\mathbb{C}^2/\mathbb{Z}_{N(1)}$ orbifold. Before computing the mass corrections, let us summarize the low-lying spectrum of the flat orbifold without deformation.

In four dimension, the rotational symmetry is Spin(4) = $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, and it is convenient to use undotted ($\mathrm{SU}(2)_L$) and dotted ($\mathrm{SU}(2)_R$) spinor indices instead of vector ones. These indices are transformed to each other with matrices $(\sigma^m)^a{}_{\dot{b}}$ and their hermitian conjugate $(\overline{\sigma}^m)^{\dot{a}}{}_b$. The Spin(4) currents $J^{mn}$ are decomposed into $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ parts with two spinor indices by

$$\frac{1}{2}J_{mn}(\sigma^m)^a{}_{\dot{b}}(\overline{\sigma}^n)^{\dot{a}}{}_b = J^a{}_b\delta^{\dot{a}}_{\dot{b}} + J^{\dot{a}}{}_{\dot{b}}\delta^a_b\,. \tag{3.1}$$

With this definition, the current with spinor indices $J^{12}$ is a hermitian operator and eigenvalues of its zero-mode $J_0^{12}$ for $\mathrm{SU}(2)_L$ doublet are $\pm 1/2$. The $\mathrm{SU}(2)_R$ current $J^{\dot{1}\dot{2}}$ is also normalized in the same way. The target space coordinates $\Phi^m$ and their superpartners $\Psi^m$ are also rewritten as bi-spinors $\Phi^{a\dot{a}}$ and $\Psi^{a\dot{a}}$. We consider $\mathbb{C}^2/\mathbb{Z}_N$ orbifold with the $\mathbb{Z}_N$ generator $\Gamma$ defined by

$$\Gamma = \exp\left(\frac{4\pi i}{N}(J_0^{\dot{1}\dot{2}})\right). \tag{3.2}$$

In this orbifold, half of supersymmetries belonging to $\mathbf{2}_R$ are broken and the other half in $\mathbf{2}_L$ remains unbroken. ($\mathbf{2}_L$ and $\mathbf{2}_R$ denote $(\mathbf{2},\mathbf{1})$ and $(\mathbf{1},\mathbf{2})$ representations of $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$, respectively.) There are $N-1$ twisted sectors labeled by a non-zero $\mathbb{Z}_N$ integer $w$. We also use a parameter $\kappa = w/N$ instead of $w$. We consider the NS-NS sector of a twisted mode with boundary condition

$$\Phi^{a\dot{1}}(\sigma+2\pi) = e^{2\pi i\kappa}\Phi^{a\dot{1}}(\sigma)\,, \qquad \Psi^{a\dot{1}}(\sigma+2\pi) = -e^{2\pi i\kappa}\Psi^{a\dot{1}}(\sigma)\,. \tag{3.3}$$

$\Phi^{a\dot{2}}$ and $\Psi^{a\dot{2}}$ are expressed with $\Phi^{a\dot{1}}$ and $\Psi^{a\dot{1}}$ by the hermiticity conditions $\epsilon_{ac}\epsilon_{\dot{b}\dot{d}}\Phi^{c\dot{d}} = (\Phi^{a\dot{b}})^\dagger$ and $\epsilon_{ac}\epsilon_{\dot{b}\dot{d}}\Psi^{c\dot{d}} = (\Psi^{a\dot{b}})^\dagger$. This theory can be quantized in the ordinary way in the light-cone formalism. Let us assume $0 < \kappa \le 1/2$. The free part of the hamiltonian $H_0 = L_0 + \widetilde{L}_0$ is given by

$$L_0 = N - \frac{1}{2} + \kappa\,, \qquad \widetilde{L}_0 = \widetilde{N} - \frac{1}{2} + \kappa\,. \tag{3.4}$$

We are interested only in massless and tachyonic states. To make such states, only creation oscillators we can act on the vacuum state $|0\rangle_L \times |0\rangle_R$ are the following four.

$$\psi^{a\dot{2}}_{-1/2+\kappa}\,, \qquad \widetilde{\psi}^{a\dot{1}}_{-1/2+\kappa}\,, \qquad \phi^{a\dot{1}}_{-\kappa}\,, \qquad \widetilde{\phi}^{a\dot{2}}_{-\kappa}\,. \tag{3.5}$$

We have additional fermionic zero modes for $\kappa = 1/2$. In the supersymmetric case, however, we can obtain massless spectrum in the $\kappa = 1/2$ case as a limit of $\kappa < 1/2$ case and we do not have to treat it separately. This is not the case for non-supersymmetric orbifold we will discuss later.

In the case of supersymmetric orbifold, both $|0\rangle_L$ and $|0\rangle_R$ are GSO odd, and we need at least one fermionic oscillators in both the left and the right moving parts. We have four degenerate massless states.

$$|ab\rangle \equiv \psi^{a\dot{2}}_{-1/2+\kappa}|0\rangle_L \otimes \widetilde{\psi}^{b\dot{1}}_{-1/2+\kappa}|0\rangle_R\,. \tag{3.6}$$

These states carry $K_0^{\dot{1}\dot{2}} = \widetilde{K}_0^{\dot{1}\dot{2}} = 0$ and are invariant under $\Gamma$. We recombine $|ab\rangle$ into the $\mathrm{SU}(2)_L$ singlet $|\mathbf{1}\rangle$ and the triplet $|\mathbf{3}(m)\rangle$ where $m = 0, \pm 1$ represents the $J^{12}$ eigenvalue. The triplet states for each $w$ corresponds to blow-up modes associated with one of $\mathbf{S}^2$ cycles of $A_{N-1}$ singularity, while the singlet mode for each $w$ is related to the $B$-field integrated over each two cycle.

Let us now compute the mass correction to each state we have obtained above. In four dimension, the Weyl tensor has 10 independent components belonging to $\mathbf{5}_L + \mathbf{5}_R$. This can be easily checked as follows. Because we can regard each pair of indices $mn$ and $pq$ of the curvature tensor $R_{mnpq}$ as an index for the adjoint representation of $\mathrm{Spin}(4) = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, $R_{mnpq}$ belongs to the symmetric product of two adjoint representations $[(\mathbf{3}_L + \mathbf{3}_R) \times (\mathbf{3}_L + \mathbf{3}_R)]_{\mathrm{sym}} = \mathbf{5}_L + \mathbf{5}_R + \mathbf{3}_L \times \mathbf{3}_R + \mathbf{1} + \mathbf{1}$. One of the singlets and $\mathbf{3}_L \times \mathbf{3}_R$ correspond to the ten components of the Ricci tensor $R_{mn}$, which is assumed to vanish. The other singlet represents $\epsilon^{mnpq} R_{mnpq}$, which also vanishes owing to the Bianchi identity. Thus, we have only ten independent components of the curvature tensor belonging to $\mathbf{5}_L + \mathbf{5}_R$. We can extract $\mathbf{5}_L$ components $R_{abcd}$ and $\mathbf{5}_R$ components $R_{\dot{a}\dot{b}\dot{c}\dot{d}}$ as

$$\frac{1}{4} R_{mnpq} (\sigma^m)^a{}_{\dot{a}} (\overline{\sigma}^n)^{\dot{b}}{}_b (\sigma^p)^c{}_{\dot{c}} (\overline{\sigma}^q)^{\dot{d}}{}_d = R^a{}_b{}^c{}_d \delta^{\dot{b}}_{\dot{a}} \delta^{\dot{d}}_{\dot{c}} + R_{\dot{a}}{}^{\dot{b}}{}_{\dot{c}}{}^{\dot{d}} \delta^a_b \delta^c_d \,. \tag{3.7}$$

Both $R_{abcd}$ and $R_{\dot{a}\dot{b}\dot{c}\dot{d}}$ are completely symmetric in all of their indices. In terms of these tensors with spinor indices, the interaction term (2.9) is rewritten as

$$H_{\mathrm{int}} = -\alpha' R_{abcd} K_0^{ab} \widetilde{K}_0^{cd} - \alpha' R_{\dot{a}\dot{b}\dot{c}\dot{d}} K_0^{\dot{a}\dot{b}} \widetilde{K}_0^{\dot{c}\dot{d}} \,. \tag{3.8}$$

If we turn on only the $R_{\dot{a}\dot{b}\dot{c}\dot{d}}$ with keeping $R_{abcd} = 0$, the $\mathrm{SU}(2)_L$ part of the holonomy remains trivial. This implies that this deformation does not break supersymmetry. Therefore, we expect the states $|ab\rangle$ remain massless. Indeed, because the second term on the right hand side in (3.8) consists of $\mathrm{SU}(2)_R$ generators, which act trivially on the massless states $|ab\rangle$, all the $H_{\mathrm{int}}$ eigenvalues for $|ab\rangle$ are zero.

On the other hand, if we turn on the $\mathbf{5}_L$ part of the curvature $R_{abcd}$, the $\mathrm{SU}(2)_L$ holonomy becomes nontrivial and supersymmetry is completely broken. Although we can compute correction in general case, let us consider deformation keeping $\mathrm{U}(1)_L$ isometry for simplicity. If we impose this isometry on the deformation, only the component $R_{1122}$ of the curvature tensor is allowed to be non-zero. If it is negative, we can extend this deformation to the whole $\mathbb{C}^2$ such that the space becomes Taub-NUT manifold. The relation between the asymptotic radius $R_{\mathrm{TN}}$ of the Taub-NUT manifold and the curvature at the center is

$$R_{1122} = -\frac{4}{R_{\mathrm{TN}}^2} \,. \tag{3.9}$$

The interaction induced by this deformation is

$$H_{\mathrm{int}} = \frac{4\alpha'}{R_{\mathrm{TN}}^2} \left( 4K_0^{12} \widetilde{K}_0^{12} + K_0^{11} \widetilde{K}_0^{22} + K_0^{22} \widetilde{K}_0^{11} \right) . \tag{3.10}$$

We can easily see that $H_{\mathrm{int}}$ is diagonal on states $|\mathbf{1}\rangle$ and $|\mathbf{3}(m)\rangle$. Because $H_{\mathrm{int}}$ is written in terms of the $\mathrm{SU}(2)_L$ generators, it trivially vanishes on the singlet state $|\mathbf{1}\rangle$.

$$H_{\mathrm{int}} |\mathbf{1}\rangle = 0 \,, \tag{3.11}$$

As we mentioned before, $|\mathbf{1}\rangle$ corresponds to the $B$-field integrated over a two-cycle. We can interpret (3.11) as a result of the $B$-field gauge invariance. The eigenvalue of $H_{\rm int} \sim (\alpha'/2)\delta m^2$ on each state in the triplet is easily obtained as

$$H_{\rm int}|\mathbf{3}(0)\rangle = -\frac{8\alpha'}{R_{\rm TN}^2}|\mathbf{3}(0)\rangle\,, \qquad H_{\rm int}|\mathbf{3}(\pm1)\rangle = \frac{4\alpha'}{R_{\rm TN}^2}|\mathbf{3}(\pm1)\rangle\,. \tag{3.12}$$

Therefore, only one of the triplet modes becomes tachyonic.

### 3.2 $\mathbb{C}^3/\mathbb{Z}_3$ and $\mathbb{C}^4/\mathbb{Z}_4$

In the same way as the four dimensional case, we can compute the mass correction for other dimensions. Here we explicitly calculate the mass corrections to massless twisted modes of supersymmetric orbifolds $\mathbb{C}^3/\mathbb{Z}_3$ and $\mathbb{C}^4/\mathbb{Z}_4$.

We choose the complex coordinates $(z^1, z^2, z^3)$ of $\mathbb{C}^3$ to be $z^1 = x^1 + ix^2$, $z^2 = x^3 + ix^4$, $z^3 = x^5 + ix^6$. The orbifold is constructed by dividing this $\mathbb{C}^3$ by $\mathbb{Z}_3$ action. The generator $\Gamma$ of $\mathbb{Z}_3$ is expressed as

$$\Gamma = (-1)^{F_s}\exp\left(\frac{2\pi}{3}J_0^{12} + \frac{2\pi}{3}J_0^{34} + \frac{2\pi}{3}J_0^{56}\right), \tag{3.13}$$

where $F_s$ is the spacetime fermion number. There are two twisted sectors corresponding to nontrivial elements $\Gamma$ and $\Gamma^2$ of $\mathbb{Z}_3$, and we have one massless state in each sector. We denote them by $|\mathbf{1}\rangle_\Gamma$ and $|\mathbf{1}\rangle_{\Gamma^2}$.

In six dimension the curvature tensor belongs to $\mathbf{84}_{(202)}+\mathbf{20}_{(020)}+\mathbf{1}_{(000)}$ representation of $SO(6) \simeq SU(4)$. (Here we attached the Dynkin indices to distinguish same dimensional representation.) The Ricci-flat condition removes $\mathbf{20}_{(020)}+\mathbf{1}_{(000)}$ and there remains $\mathbf{84}_{(202)}$. This $\mathbf{84}_{(202)}$ representation of SU(4) decomposes into representations of SU(3), and among them we can use $\mathbf{27}_{(22)} + \mathbf{8}_{(11)} + \mathbf{1}_{(00)}$ as the $\mathbb{Z}_3$ invariant deformations. Therefore the curvature tensor can be written explicitly as

$$\begin{aligned} R_{i\bar{j}k\bar{l}} &= R^{(27)}_{i\bar{j}k\bar{l}} + R^{(8)}_{i\bar{j}k\bar{l}} + R^{(1)}_{i\bar{j}k\bar{l}} \\ &= T^{(27)}_{ik\bar{j}\bar{l}} + \left[4\left(T^{(8)}_{i\bar{j}}\delta_{k\bar{l}} + T^{(8)}_{k\bar{l}}\delta_{i\bar{j}}\right) - \left(T^{(8)}_{i\bar{l}}\delta_{k\bar{j}} + T^{(8)}_{k\bar{j}}\delta_{i\bar{l}}\right)\right] + \\ &\quad + T^{(1)}[5\delta_{i\bar{j}}\delta_{k\bar{l}} + \delta_{i\bar{l}}\delta_{k\bar{j}}], \end{aligned} \tag{3.14}$$

where $i, j, \ldots$ $(\bar{\imath}, \bar{\jmath}, \ldots)$ denote holomorphic (anti-holomorphic) indices of $\mathbf{3}_{(10)}$ $(\bar{\mathbf{3}}_{(01)})$ representation. The tensor $T^{(27)}_{ij\bar{k}\bar{l}}$ belongs to $\mathbf{27}_{(22)}$ and satisfies

$$T^{(27)}_{ij\bar{k}\bar{l}} = T^{(27)}_{ji\bar{k}\bar{l}} = T^{(27)}_{ij\bar{l}\bar{k}}\,, \qquad T^{(27)}_{ij\bar{k}\bar{l}}\delta^{i\bar{k}} = 0\,. \tag{3.15}$$

Similarly $T^{(8)}_{i\bar{j}}$ is $\mathbf{8}_{(11)}$ representation tensor satisfying $T^{(8)}_{i\bar{j}}\delta^{i\bar{j}} = 0$. $T^{(1)}$ represents the singlet degree of freedom. Using Bianchi identity, we can calculate $R_{ik\bar{j}\bar{l}}$ component from (3.14), i.e. $R_{ik\bar{j}\bar{l}} = -R_{i\bar{j}\bar{l}k} - R_{i\bar{l}k\bar{j}}$. We can determine whether the deformation preserves Kähler property or not depending on whether $R_{ik\bar{j}\bar{l}}$ equals zero or not. Actually we can see that $\mathbf{27}_{(22)}$ deformation preserves Kähler property. On the other hand an orbifold deformed by $\mathbf{8}_{(11)}$ or $\mathbf{1}_{(00)}$ components is not Kähler and all the supersymmetries are broken in it.

As we have already obtained the explicit formula (2.9) for general deformations, we can easily calculate the mass corrections to $|\mathbf{1}\rangle_\Gamma$ and $|\mathbf{1}\rangle_{\Gamma^2}$. Because these massless states are SU(3) singlets, non-zero mass corrections come from the trace part of the generators $K^{l\bar{m}}$ and $\widetilde{K}^{l\bar{m}}$:

$$\langle K^{l\bar{m}}\rangle_\Gamma = \langle \widetilde{K}^{l\bar{m}}\rangle_{\Gamma^2} = \delta^{l\bar{m}}\,, \qquad \langle \widetilde{K}^{l\bar{m}}\rangle_\Gamma = \langle K^{l\bar{m}}\rangle_{\Gamma^2} = -\delta^{l\bar{m}}\,. \tag{3.16}$$

As a result we obtain the following mass correction to the massless states:

$$\frac{\alpha'}{2}m^2 = H_{\rm int} = 48\alpha' T^{(1)}\,. \tag{3.17}$$

For $\mathbb{C}^4/\mathbb{Z}_4$ orbifold, we choose the complex coordinates in the similar way as $\mathbf{C^3}$ case ($z^1 = x^1 + ix^2, \ldots$) and the action of the generator $\Gamma$ to be

$$\Gamma = (-1)^{F_s}\exp\left(\frac{2\pi}{4}J_0^{12} + \frac{2\pi}{4}J_0^{34} + \frac{2\pi}{4}J_0^{56} + \frac{2\pi}{4}J_0^{78}\right), \tag{3.18}$$

so that this orbifold is supersymmetric. There are two massless states in $\Gamma$ and $\Gamma^3$ twisted sectors. (All the $\Gamma^2$ twisted modes are massive.) The eight dimensional curvature tensor belongs to $\mathbf{300}_{(000;2)} + \mathbf{35}_{(200;0)} + \mathbf{1}_{(000;0)}$ representation of SO(8). (Here we used the convention in which the first three Dynkin indices are related by triality.) Among them, Ricci-flat curvature tensor belongs to $\mathbf{300}_{(000;2)}$. Decomposing $\mathbf{300}_{(000;2)}$ into SU(4) representation and picking up $\mathbb{Z}_4$ invariant ones, we obtain $\mathbf{84}_{(202)} + \mathbf{20}_{(020)} + \mathbf{15}_{(101)} + \mathbf{1}_{(000)}$. The curvature tensor can be written as

$$\begin{aligned} R_{i\bar{j}k\bar{l}} &= R^{(84)}_{i\bar{j}k\bar{l}} + R^{(20)}_{i\bar{j}k\bar{l}} + R^{(15)}_{i\bar{j}k\bar{l}} + R^{(1)}_{i\bar{j}k\bar{l}} \\ &= T^{(84)}_{ik\bar{j}\bar{l}} + T^{(20)}_{ik\bar{j}\bar{l}} + \left[T^{(15)}_{i\bar{j}}\delta_{k\bar{l}} + T^{(15)}_{k\bar{l}}\delta_{i\bar{j}}\right] + T^{(1)}[7\delta_{i\bar{j}}\delta_{k\bar{l}} + 2\delta_{i\bar{l}}\delta_{k\bar{j}}]. \end{aligned} \tag{3.19}$$

$T^{(84)}_{ij\bar{k}\bar{l}}$ satisfies the same condition as $T^{(27)}_{ij\bar{k}\bar{l}}$ in (3.15). $T^{(20)}_{ij\bar{k}\bar{l}}$ satisfies the condition $T^{(20)}_{ij\bar{k}\bar{l}} = -T^{(20)}_{ji\bar{k}\bar{l}} = -T^{(20)}_{ij\bar{l}\bar{k}}$ and $T^{(20)}_{ij\bar{k}\bar{l}}\delta^{i\bar{k}} = 0$. $T^{(15)}_{i\bar{j}}$ tensor satisfies the same condition as $T^{(8)}_{i\bar{j}}$ in the six dimensional case; $T^{(15)}_{i\bar{j}}\delta^{i\bar{j}} = 0$. In the same way as the six dimensional case, we can determine whether each deformation preserves Kähler property. Actually only $\mathbf{84}_{(202)}$ deformation preserves Kähler property and other three deformations lead to non-Kähler manifolds.

Mass correction of massless states caused by these deformation are also easily calculated in the same way as the six dimensional case, and we obtain

$$\frac{\alpha'}{2}m^2 = H_{\rm int} = 120\alpha' T^{(1)}\,. \tag{3.20}$$

In both six and eight dimensional cases, the mass corrections (3.17) and (3.20) depend only on the singlet component of the curvature, which break all the supersymmetries. We can choose, at least locally, the signature of the deformation parameters so that these states become tachyonic, although it is not clear if we can extend these deformations to the whole space.

## 4. Non-supersymmetric $\mathbb{C}^2/\mathbb{Z}_{N(N+1)}$

As is mentioned in the introduction, one purpose of this paper is to confirm the result in [22], in which the closed string tachyon condensation on a type 0 NS5-brane in $\mathbf{S}^1$ compactified background is identified with decay of spacetime with creating a Witten's bubble. In section 4.1 we analyze the tachyonic masses including numerical factors in the type 0 side. However, as we will comment later, calculations in section 4.1 are actually quite unreliable because of the large curvature of type 0 settings and the smearing of NS5-branes. The reason why we present this analysis is to give an intuitive explanation for the existence of localized tachyons on type 0 NS5-branes. In section 4.2 we give a more reliable computation using the formula (2.9) for the deformed orbifold and then we will see agreements except some corrections due to the localization of NS5-branes.

### 4.1 Tachyonic modes on type 0 NS5-branes

Let us consider $N_{\rm NS5}$ parallel type 0 NS5-branes. Type 0 theory has the closed string tachyon field in the bulk. To localize it on NS5-branes, we compactify one direction transverse to the NS5-branes, and impose anti-periodic boundary condition on the tachyon field. If the compactification radius is sufficiently small, the bulk tachyon becomes massive due to the large Kaluza-Klein momentum. However, there are remnant of tachyonic modes near the NS5-branes as we see below [24]. We are interested in the T-duality between these NS5-branes and an orbifold. More precisely, NS5-branes dual to the orbifold we will discuss later are distributed along an $\mathbf{S}^1$ at even intervals. Furthermore, at the supergravity level, the dual configuration is the smeared NS5-brane solution

$$ds^2 = \eta_{ij}dx^i dx^j + H(x^\mu)\delta_{\mu\nu}dx^\mu dx^\nu\,, \qquad e^{2\phi} = \text{const.} \times H(x^\mu)\,, \tag{4.1}$$

where $x^i$ ($i = 0,\ldots,5$) and $x^\mu$ ($\mu = 6,7,8,9$) are parallel and transverse coordinates respectively. $x^9$ is periodic coordinate with period $2\pi$. The harmonic function $H$ is given by

$$H = \frac{\alpha' N_{\rm NS5}}{2r} + R^2\,, \qquad r^2 = (x^6)^2 + (x^7)^2 + (x^8)^2\,. \tag{4.2}$$

The parameter $R$ determine the $\mathbf{S}^1$ radius at the asymptotic region $r \to \infty$. When we regard this as the T-dual of the orbifold, we should set $R = 0$. Later we discuss the deformation of orbifold to a Kaluza-Klein monopole geometry with asymptotic radius $\widetilde{R}$. The Kaluza-Klein monopole is described as a $\mathbb{Z}_{2N_{\rm NS5}}$ orbifold of the Taub-NUT manifold with asymptotic radius $R_{\rm TN}$. The relation among these radii is

$$\frac{R_{\rm TN}}{2N_{\rm NS5}} = \widetilde{R} = \frac{\alpha'}{2R}\,. \tag{4.3}$$

We can obtain localized tachyons on NS5-branes as normalizable solutions of the equation of motion

$$\partial_i g^{ij}\partial_j T = \left(-\frac{e^{2\phi}}{\sqrt{-g}}\partial_\mu g^{\mu\nu}\frac{\sqrt{-g}}{e^{2\phi}}\partial_\nu + M_T^2\right)T\,. \tag{4.4}$$

(When we obtain this equation of motion, it is important that $(H_{\mu\nu\rho})^2T^2$ term is absent in the action of type 0 theory [25].) Therefore, the problem determining mass spectrum of localized tachyons on the NS5-branes reduces to one determining eigenvalue of the four-dimensional Klein-Gordon like operator on the right hand side in (4.4). Each eigen function factorizes into an exponential factor $e^{ikx^9}$ associated with $\mathbf{S}^1$ direction and function $\psi(x^6, x^7, x^8)$ depending on three non-compact transverse coordinates. Because the tachyon field is anti-periodic along $\mathbf{S}^1$, $k$ is a half odd integer. The differential equation satisfied by $\psi$ has the same structure with the Schrödinger equation for a particle in a Coulomb potential, and is easily solved analytically. The wave functions and corresponding eigenvalues are labeled by four quantum numbers. One is the Kaluza-Klein momentum $k$ along $\mathbf{S}^1$. The other three are the same with those for a hydrogen atom. Namely, the principal quantum number $n$, the azimuthal one $l$ and the magnetic one $m$. By solving (4.4), we obtain the following mass spectrum

$$\frac{\alpha'}{2}M_6^2 = -1 + \frac{2n|k|}{N_{\mathrm{NS5}}} - \frac{4n^2\alpha'}{R_{\mathrm{TN}}^2} + \mathcal{O}\left(\frac{\alpha'}{R_{\mathrm{TN}}^4}\right), \tag{4.5}$$

where the normalization is so chosen that this can be identified with worldsheet hamiltonian $L_0 + \widetilde{L}_0$. Just as the spectrum of a hydrogen atom, the mass eigenvalues do not depend on $l$ and $m$, and for each $k$ and $n$, there are $n^2$ degenerate states.

For $N_{\mathrm{NS5}} = 1$, the corresponding $\mathbb{C}^2/\mathbb{Z}_2$ orbifold is supersymmetric and there is no tachyonic state. Actually, the first two terms on the right hand side in (4.5) cancels each other for $(k, n) = (\pm 1/2, 1)$ (1s state). If it is deformed and $R_{\mathrm{TN}}$ becomes finite, the leading mass correction is given by the third term in (4.5). The tachyonic mass does not include string scale. (The common $\alpha'$ factors on the left and right hand sides can be removed.) This fact suggests us that we may be able to analyze the dual configuration by means of classical gravity. Indeed, it was shown in [22] that we reproduce the same tachyonic mass up to numerical factor by analyzing the unstable mode associated with the creation of Witten's bubble. In [22], only the existence of the unstable modes is shown, but the number of unstable modes and numerical values of the tachyonic masses are left undetermined. In the next section, we give the answer to this question by computing the mass correction to the twisted modes.

Before ending this subsection, we would like to emphasize the following point. Here we treated the tachyon field quantum mechanically, and neglected other stringy excitations. In fact this cannot be justified because the curvature of the background spacetime is of the order of the string scale. Furthermore, we use a smeared solution rather than a localized one. Even if the parameter $R$ is however small, using a smeared solution is not a good approximation because the proper compactification radius near the NS5-branes is not small due to the divergent metric.

In the next subsection, we reconsider the tachyonic spectrum in the dualized type II theory. There the calculations are more reliable and we obtain the correct tachyonic mass spectrum

### 4.2 Twisted mode analysis

The dual configuration to the type 0 NS5-branes discussed above is Taub-NUT deformation of non-supersymmetric $\mathbb{C}^2/\mathbb{Z}_{N(N+1)}$ orbifold with $N = 2N_{\mathrm{NS5}}$ [26, 27]. Note that $N$



JHEP11(2004)042

is always even integer. It can be realized by replacing the generator $\Gamma$ in (3.2) for the supersymmetric orbifold by

$$\Gamma = (-)^{F_s} \exp\left(\frac{4\pi i}{N}(J_0^{\dot{1}\dot{2}})\right). \tag{4.6}$$

The sectors with even $w$ are identical to the supersymmetric case which we have already discussed in the last section. We only need to consider odd $w$ sectors separately. The extra factor $(-1)^{F_s}$ in (4.6) reverses the GSO parity in the odd $w$ sectors, and both $|0\rangle_L$ and $|0\rangle_R$ become GSO even. We have to act even number of fermionic oscillators on the vacuum state in each of the left and the right moving parts.

Let us consider $0 < \kappa < 1/2$ case first and leave the $\kappa = 1/2$ case for later. If $0 < \kappa < 1/2$, the excitation by two fermionic oscillators makes the state massive. Therefore, we can only use bosonic ones in (3.5).

$$|n\rangle \equiv \phi_{-\kappa}^{a_1 \dot{1}} \cdots \phi_{-\kappa}^{a_{n-1} \dot{1}} |0\rangle_L \otimes \widetilde{\phi}_{-\kappa}^{b_1 \dot{2}} \cdots \widetilde{\phi}_{-\kappa}^{b_{n-1} \dot{2}} |0\rangle_R . \tag{4.7}$$

Each of left and right moving part has $n-1$ symmetric $\mathrm{SU}(2)_L$ indices and thus belongs to spin $(n-1)/2$ representation of $\mathrm{SU}(2)_L$. Combining the left and right moving parts, we obtain $n^2$ degenerate states. These degenerate states together are denoted by $|n\rangle$ and we will not distinguish each of them. The $\mathrm{U}(1)_R$ charges of these states are given by

$$K_0^{\dot{1}\dot{2}}|n\rangle = \frac{n}{2}|n\rangle, \qquad \widetilde{K}_0^{\dot{1}\dot{2}}|n\rangle = -\frac{n}{2}|n\rangle . \tag{4.8}$$

Therefore, $J_0^{\dot{1}\dot{2}}|n\rangle = 0$ and these states are invariant under $\Gamma$. The energy of these states are

$$(L_0 + \widetilde{L}_0)|n\rangle = (-1 + 2n\kappa)|n\rangle . \tag{4.9}$$

The states $|n\rangle$ are tachyonic if $n$ is smaller than $1/(2\kappa)$. This reproduce the spectrum in (4.5) with $R_{\mathrm{TN}} \to \infty$. (Note that $\kappa = w/N$ is identified with $k/N_{\mathrm{NS5}}$.)

Let us consider deformation of this orbifold. We restrict our attention to the Taub-NUT deformation which is dual to the type 0 NS5-branes. This deformation keeps the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_R$ isometry, and only non-vanishing component of the curvature tensor is $R_{\dot{1}\dot{1}\dot{2}\dot{2}} = -4/R_{\mathrm{TN}}^2$, where $R_{\mathrm{TN}}$ is the asymptotic radius of the Taub-NUT manifold. When $\kappa \neq 1/2$, currents $J^{\dot{1}\dot{1}}$ and $J^{\dot{2}\dot{2}}$ do not commute with $\Gamma^w \in \mathrm{U}(1)_R$ and their zero modes are absent. The hamiltonian is simplified to

$$H_{\mathrm{int}} = -4\alpha' R_{\dot{1}\dot{1}\dot{2}\dot{2}} K_0^{\dot{1}\dot{2}} \widetilde{K}_0^{\dot{1}\dot{2}} , \tag{4.10}$$

and the mass correction to $|n\rangle$ is easily obtained as

$$H_{\mathrm{int}}|n\rangle = \alpha' n^2 R_{\dot{1}\dot{1}\dot{2}\dot{2}}|n\rangle = -\frac{4n^2\alpha'}{R_{\mathrm{TN}}^2}|n\rangle . \tag{4.11}$$

This precisely reproduce the third term in (4.5), obtained by solving the Klein-Gordon equation on the smeared NS5-brane solution.



JHEP11(2004)042

In the $\kappa = 1/2$ case, which occurs if $N = 2 \mod 4$, there are additional massless states. Let us look at a state $\psi^{1\dot{2}}_{-1/2+\kappa}\psi^{2\dot{2}}_{-1/2+\kappa}|0\rangle_L$ in the left moving part. The energy of this state $L_0 = 1/2 - \kappa$ vanishes when $\kappa = 1/2$. This massless state makes $\mathrm{SU}(2)_R$ doublet together with another massless state $|0\rangle_L$. Therefore, combining with right moving part, which is also $\mathrm{SU}(2)_R$ doublet in the same manner, we have four states belonging to $\mathbf{2}_R \times \mathbf{2}_R$. We decompose them to the $\mathrm{SU}(2)_R$ singlet $|\dot{\mathbf{1}}\rangle$ and the $\mathrm{SU}(2)_R$ triplet $|\dot{\mathbf{3}}(m)\rangle$ just as in the supersymmetric case. In the $N = 2$ case, the orbifold $\mathbb{C}^2/\mathbb{Z}_{2(2+1)}$ is accidentally supersymmetric, and is the parity transformation of $\mathbb{C}^2/\mathbb{Z}_{2(1)}$, which we studied in the previous section. We can identify $|\dot{\mathbf{3}}(m)\rangle$ and $|\dot{\mathbf{1}}\rangle$ with the blow-up modes of $A_1$ singularity and the $B$-field integrated over the 2-cycle, respectively.

For the $\kappa = 1/2$ sector, Spin(4) symmetry is recovered and, in addition to $K^{\dot{1}\dot{2}}$ and $\widetilde{K}^{\dot{1}\dot{2}}$, four charges $K_0^{\dot{1}\dot{1}}$, $K_0^{\dot{2}\dot{2}}$, $\widetilde{K}_0^{\dot{1}\dot{1}}$, and $\widetilde{K}_0^{\dot{2}\dot{2}}$ contribute to the mass correction.

$$H_{\mathrm{int}} = -\alpha' R_{\dot{1}\dot{1}\dot{2}\dot{2}} \left(4K_0^{\dot{1}\dot{2}}\widetilde{K}_0^{\dot{1}\dot{2}} + K_0^{\dot{1}\dot{1}}\widetilde{K}_0^{\dot{2}\dot{2}} + K_0^{\dot{2}\dot{2}}\widetilde{K}_0^{\dot{1}\dot{1}}\right) \tag{4.12}$$

The matrix elements of $H_{\mathrm{int}}$ can be computed in the same way with $\mathbf{5}_L$ deformation of the supersymmetric orbifold.

$$H_{\mathrm{int}}|\dot{\mathbf{1}}\rangle = 0\,, \qquad H_{\mathrm{int}}|\dot{\mathbf{3}}(0)\rangle = -\frac{8\alpha'}{R_{\mathrm{TN}}^2}|\dot{\mathbf{3}}(0)\rangle\,, \qquad H_{\mathrm{int}}|\dot{\mathbf{3}}(\pm 1)\rangle = \frac{4\alpha'}{R_{\mathrm{TN}}^2}|\dot{\mathbf{3}}(\pm 1)\rangle\,. \tag{4.13}$$

We can understand the masslessness of the singlet state as a result of the $B$-field gauge symmetry. There is only one tachyonic state, which carries no $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ charges, and it is nothing but the unstable mode associated with the creation of a Witten's bubble discussed in [22]. We have now confirmed that this is the unique unstable mode and obtain the mass including the numerical factor, which was not obtained in [22].

Let us compare the mass correction (4.13) obtained here to (4.5). In the previous subsection, we only considered Kaluza-Klein modes without windings. Therefore, we should compare them to twisted modes without Kaluza-Klein momentum $J_0^{\dot{1}\dot{2}} = K_0^{\dot{1}\dot{2}} + \widetilde{K}_0^{\dot{1}\dot{2}}$. Among four states, we have two such states $|\dot{\mathbf{1}}\rangle$ and $|\dot{\mathbf{3}}(0)\rangle$. One is massless and the other is tachyonic. On the other hand, from the equation (4.5), we expect the emergence of two degenerate tachyonic modes with quantum numbers $(k, n) = (\pm 1/2, 1)$. This discrepancy is explained as follows. We used the smeared NS5-brane solution in section 4.1 to obtain (4.5). It gives only momentum $P$ preserving amplitude correctly. The NS5-brane is actually localized at a point on the circle, and it causes $P$-violating scattering. The momentum operator $P$ in the type 0 side is related to Spin(4) charges by $P = K_0^{\dot{1}\dot{2}} - \widetilde{K}_0^{\dot{1}\dot{2}}$, which does not commute with $H_{\mathrm{int}}$ in (4.12). If we ignore the $P$-violating terms in (4.12), the interaction hamiltonian reduces to (4.10), which we used for $\kappa \neq 1/2$, and we obtain the degenerate tachyonic mass obtained in section 4.1. Therefore, we conclude that the discrepancy between Klein-Gordon equation analysis and the twisted mode analysis is due to the ignorance of the localization of NS5-branes. One may expect that we can obtain the correct spectrum if we take account of the localization of NS5-brane and solve the Klein-Gordon equation on the geometry. However, we can see that this is not the case as long as we use the field theory approximation. The field theory approximation for the



JHEP11(2004)042

tachyon field seems not to work in this situation and we probably need to use string theory including excitation modes in the localized NS5-brane background in order to obtain the masses correctly.

## 5. Conclusions and discussions

In this paper, we computed the mass correction to the twisted modes in deformed orbifolds for several concrete examples. We found that we can obtain localized tachyons with masses of order of the curvature turned on if all the supersymmetries are broken on the deformed orbifold.

The emergence of tachyonic modes indicates the instability of the spacetime. Indeed, the tachyonic mode of the deformed $\mathbb{C}^2/\mathbb{Z}_2$ obtained in section 4.2 corresponds to the instability of the spacetime against the creation of the Witten's bubble studied in [22]. The result in section 4.2 confirms that this is only the unstable.

Although we discussed the relation between deformed orbifold and dual NS5-brane configurations only for non-supersymmetric $\mathbb{C}^2/\mathbb{Z}_{2N_{\rm NS5}(2N_{\rm NS5}+1)}$ case, it may also be interesting to study dual descriptions of other unstable orbifold deformations.

The number of localized tachyons depends on the choice of an orbifold and the way of deformation. It may be possible to construct solitonic objects with these tachyons. In the $\mathbb{C}^2/\mathbb{Z}_2$ case, which we studied in detail, there is one localized tachyon. Unfortunately, we cannot use this tachyon field to construct a domain wall. The moduli space associated with three blow-up modes of $\mathbb{C}^2/\mathbb{Z}_2$ is $\mathbb{R}^3/\mathbb{Z}_2$. Although one of three axes in $\mathbb{R}^3$ becomes tachyonic by the deformation making $R_{\rm TN}$ finite, the positive and the negative directions of the axis are identified by $\mathbb{Z}_2$. Because of this, we have only one direction along which the tachyon condensates and two different domains cannot be produced. Thus we cannot use this tachyon to make domain walls. However, we may be able to obtain localized tachyons available for the construction of solitonic objects by different choices of orbifolds and deformations. The localized tachyons obtained in the $\mathbb{C}^3/\mathbb{Z}_3$ and $\mathbb{C}^4/\mathbb{Z}_4$ cases are potential candidates. We hope to come back to the subject elsewhere.

## Acknowledgments

We thank T. Kawano, Y. Nakayama, Y. Tachikawa, H. Takayanagi for useful discussions. Y.I. is supported in part by Grant-in-Aid for the Encouragement of Young Scientists (#15740140) from the Japan Ministry of Education, Culture, Sports, Science and Technology, and by Rikkyo University Special Fund for Research.

## References

[1] A. Sen, *Stable non-BPS bound states of BPS D-branes*, *J. High Energy Phys.* **08** (1998) 010 [hep-th/9805019].

[2] A. Sen, *Tachyon condensation on the brane antibrane system*, *J. High Energy Phys.* **08** (1998) 012 [hep-th/9805170].

[3] A. Sen, SO(32) *spinors of type-I and other solitons on brane-antibrane pair*, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141].

[4] A. Sen, *Descent relations among bosonic D-branes*, *Int. J. Mod. Phys.* **A 14** (1999) 4061 [hep-th/9902105].

[5] M.S. Costa and M. Gutperle, *The Kaluza-Klein Melvin solution in M-theory*, *J. High Energy Phys.* **03** (2001) 027 [hep-th/0012072].

[6] M. Gutperle and A. Strominger, *Fluxbranes in string theory*, *J. High Energy Phys.* **06** (2001) 035 [hep-th/0104136].

[7] A. Adams, J. Polchinski and E. Silverstein, *Don't panic! closed string tachyons in ALE space-times*, *J. High Energy Phys.* **10** (2001) 029 [hep-th/0108075].

[8] A.A. Tseytlin, *Magnetic backgrounds and tachyonic instabilities in closed string theory*, in proceedings of the *10-th Tohwa International Symposium on String Theory*, Fukuoka, Japan, July 3-7, 2001 [hep-th/0108140].

[9] A. Dabholkar, *On condensation of closed-string tachyons*, *Nucl. Phys.* **B 639** (2002) 331 [hep-th/0109019].

[10] C. Vafa, *Mirror symmetry and closed string tachyon condensation*, hep-th/0111051.

[11] J.A. Harvey, D. Kutasov, E.J. Martinec and G.W. Moore, *Localized tachyons and RG flows*, hep-th/0111154.

[12] Y. Michishita and P. Yi, *D-brane probe and closed string tachyons*, *Phys. Rev.* **D 65** (2002) 086006 [hep-th/0111199].

[13] J.R. David, M. Gutperle, M. Headrick and S. Minwalla, *Closed string tachyon condensation on twisted circles*, *J. High Energy Phys.* **02** (2002) 041 [hep-th/0111212].

[14] S. Nam and S.-J. Sin, *Condensation of localized tachyons and spacetime supersymmetry*, *J. Korean Phys. Soc.* **43** (2003) 34 [hep-th/0201132].

[15] S.P. De Alwis and A.T. Flournoy, *Closed string tachyons and semi-classical instabilities*, *Phys. Rev.* **D 66** (2002) 026005 [hep-th/0201185].

[16] R. Rabadán and J. Simon, *M-theory lift of brane-antibrane systems and localised closed string tachyons*, *J. High Energy Phys.* **05** (2002) 045 [hep-th/0203243].

[17] A.M. Uranga, *Localized instabilities at conifolds*, hep-th/0204079.

[18] B.C. Da Cunha and E.J. Martinec, *Closed string tachyon condensation and worldsheet inflation*, *Phys. Rev.* **D 68** (2003) 063502 [hep-th/0303087].

[19] M. Headrick, S. Minwalla and T. Takayanagi, *Closed string tachyon condensation: an overview*, *Class. and Quant. Grav.* **21** (2004) S1539 [hep-th/0405064].

[20] D.R. Morrison, K. Narayan and M.R. Plesser, *Localized tachyons in* $\mathbb{C}^3/\mathbb{Z}^N$, *J. High Energy Phys.* **08** (2004) 047 [hep-th/0406039].

[21] Y. Okawa and B. Zwiebach, *Twisted tachyon condensation in closed string field theory*, *J. High Energy Phys.* **03** (2004) 056 [hep-th/0403051].

[22] Y. Imamura, *Decay of type 0 NS5-branes to nothing*, *Phys. Rev.* **D 69** (2004) 026005 [hep-th/0309024].

[23] M.R. Douglas and G.W. Moore, *D-branes, quivers and ALE instantons*, hep-th/9603167.


JHEP11(2004)042

[24] Y. Imamura, *Tachyonic modes on type 0 NS5-branes*, *J. High Energy Phys.* **01** (2000) 018 [hep-th/9910260].

[25] M.R. Garousi, *S-matrix elements and closed string tachyon couplings in type 0 theory*, hep-th/0309028.

[26] O. Bergman and M.R. Gaberdiel, *Dualities of type 0 strings*, *J. High Energy Phys.* **07** (1999) 022 [hep-th/9906055].

[27] Y. Imamura, *Branes in type 0/type-II duality*, *Prog. Theor. Phys.* **102** (1999) 859 [hep-th/9906090].

JHEP11(2004)042